Measures of spread continued

Overview

Quick review:

- Statistics for central tendency: mean and median
- Outliers
- Standard deviations

Z-scores

Percentiles

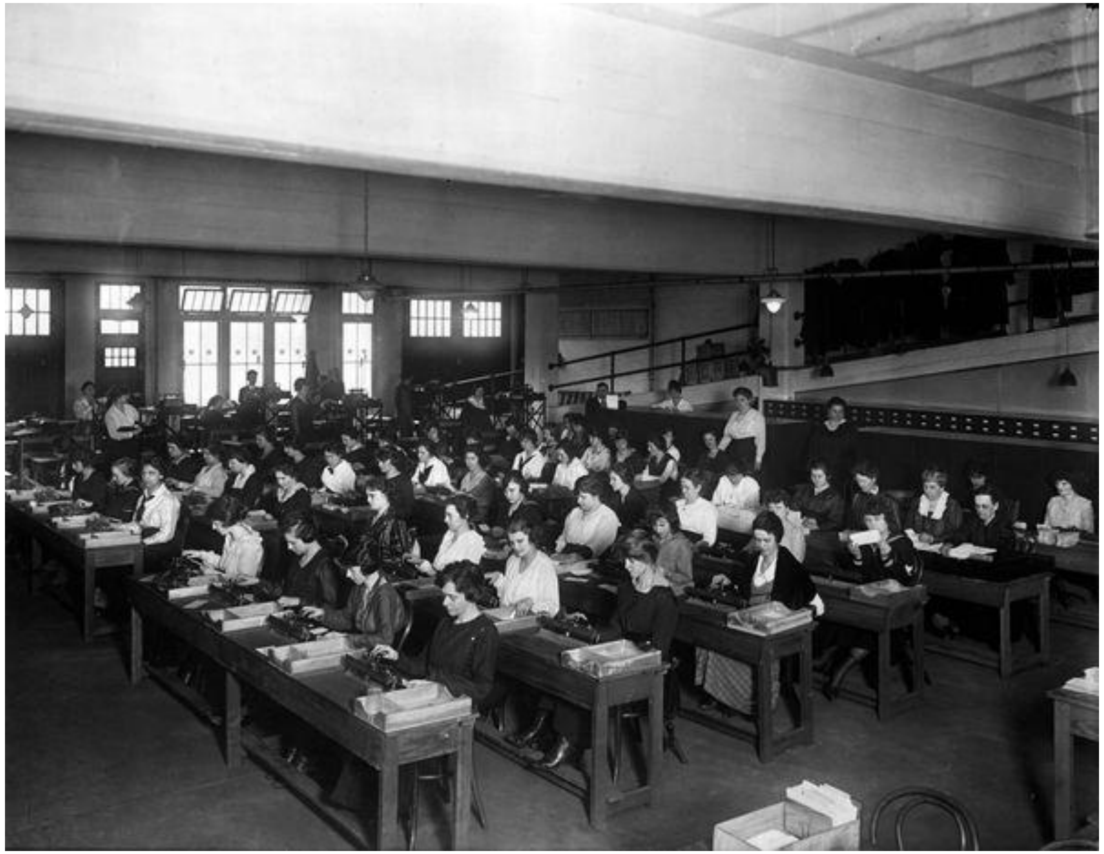
Boxplots

How did Worksheet 2 go?

Questions?

[The grinch doing yoga]

Human computers



Descriptive statistics for one quantitative variable

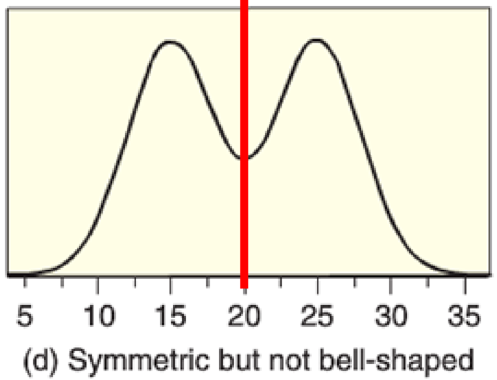
We will be looking at:

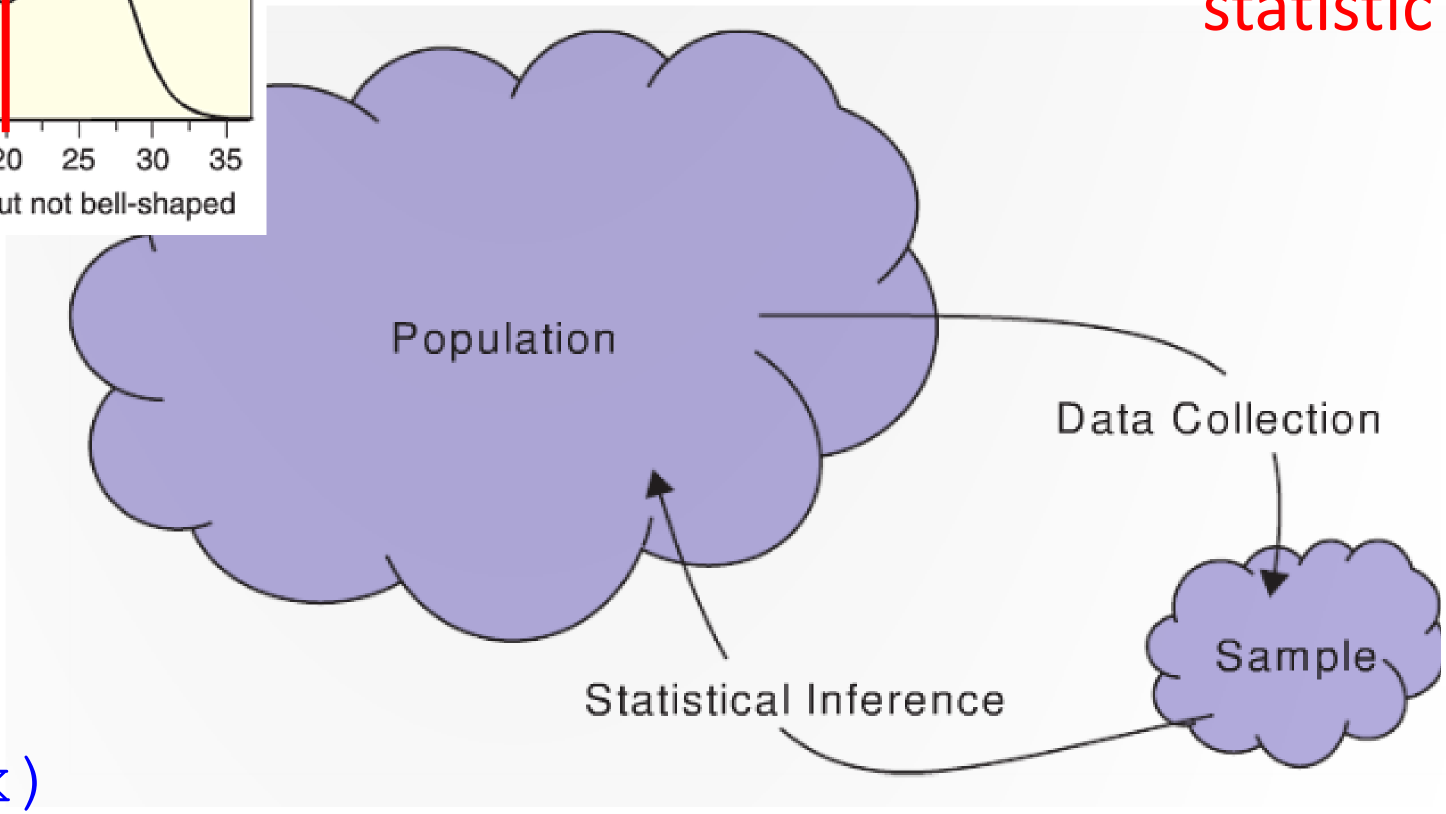
- What is the general ‘shape’ of the data
- Where are the values centered   (central tendency)
- How do the data vary   (spread)

There are all properties of how the data is ***distributed***

Central Tendency: sample and population mean

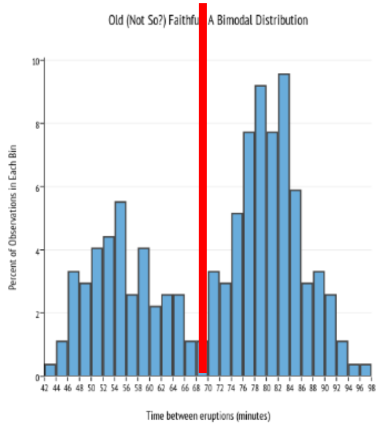
μ ← parameter





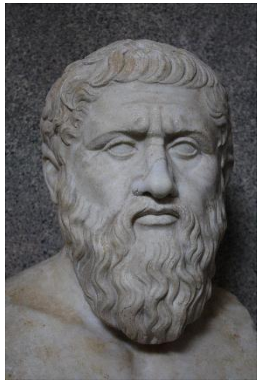
$$\bar{x} = \frac{\Sigma_i^n x_i}{n}$$

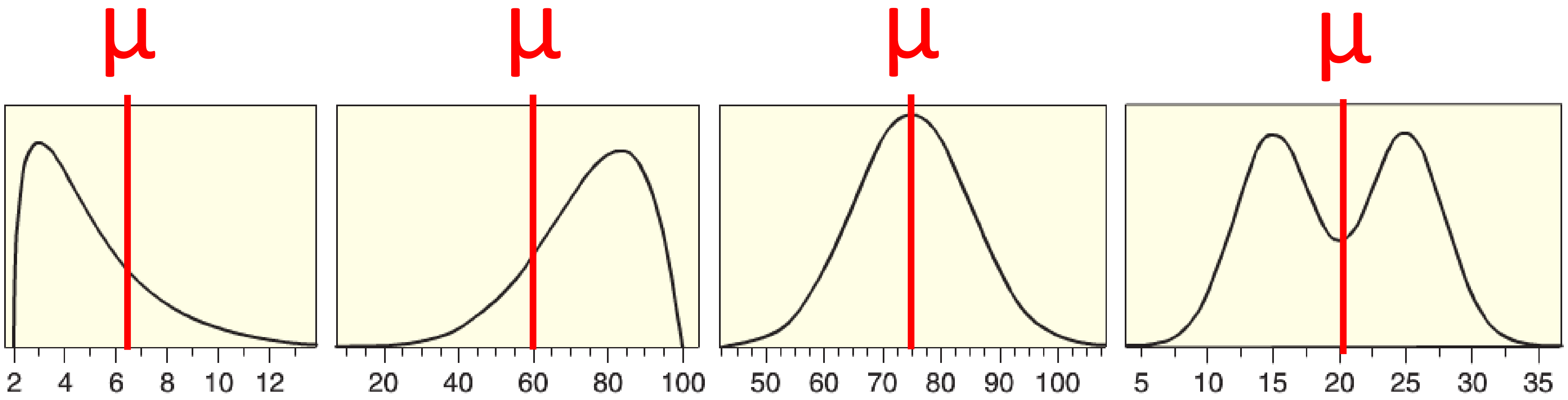
statistic → $\bar{x}$



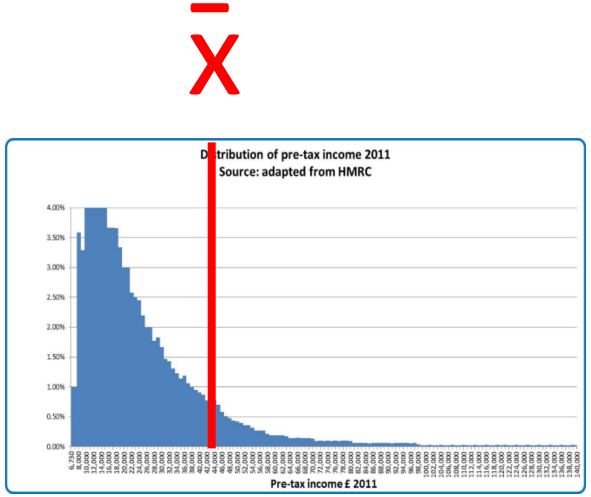
R: `mean(x)`

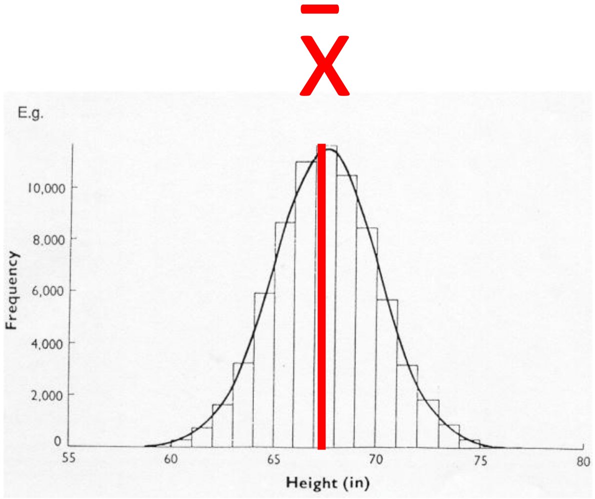
Histograms: plotting the shape of quantitative data

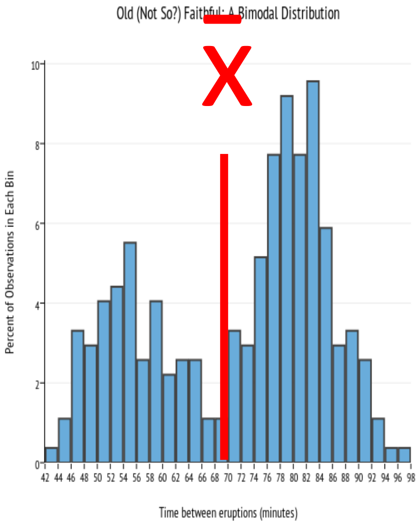




(a) Skewed to the right (b) Skewed to the left (c) Symmetric and bell-shaped (d) Symmetric but not bell-shaped



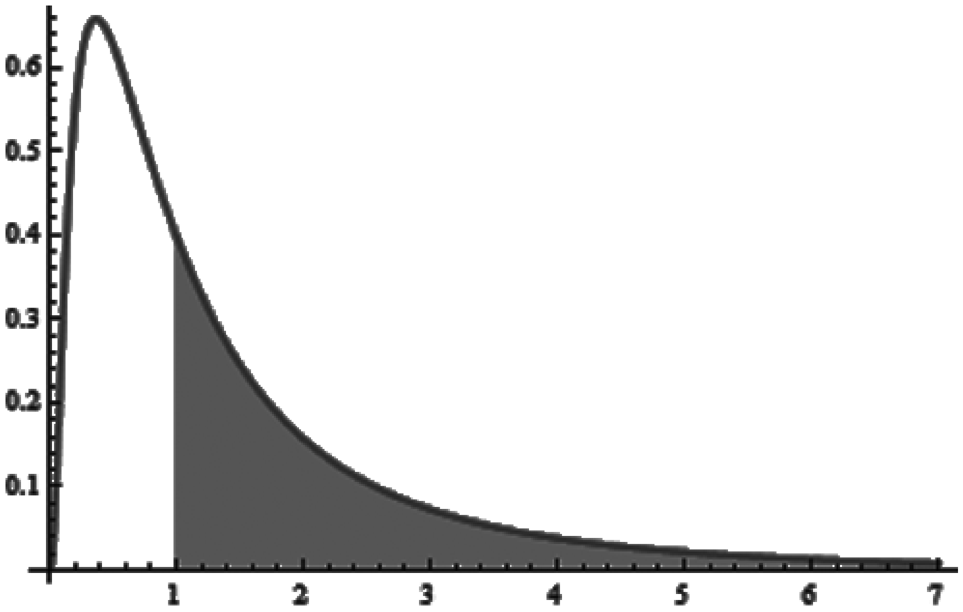


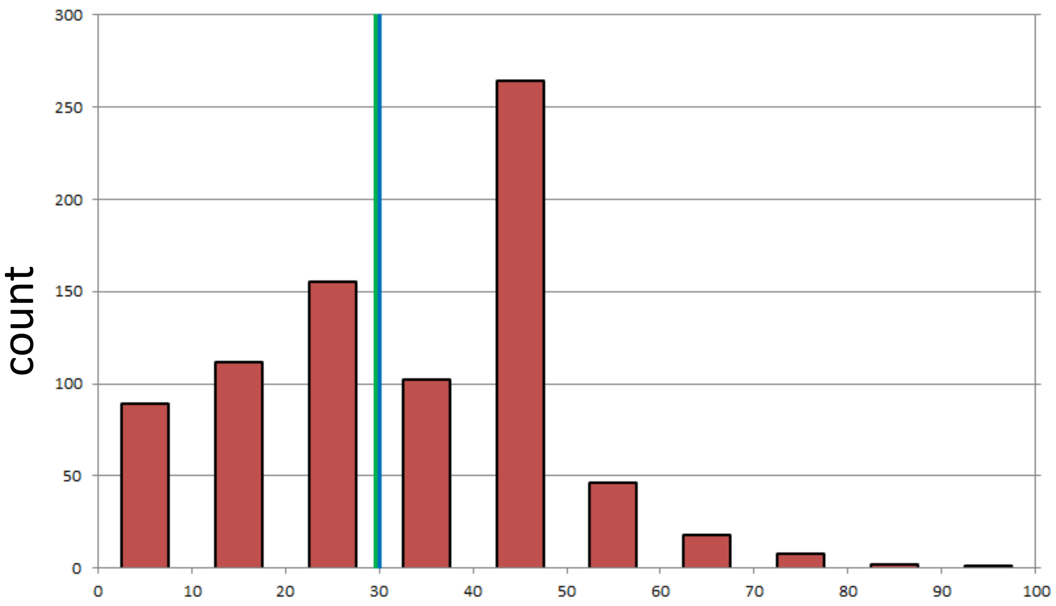


The median: another measure of central tendency

The **median** is a value that splits the data in half

- Sort the data and take the middle value (or the mean of the middle 2 values)





R: `median(v)`

Outliers

What is an **outlier?**

- A: An observed value that is notably distinct from the other values in a dataset

What are they problematic?

- A: can potentially have a large influence on the statistics you calculate

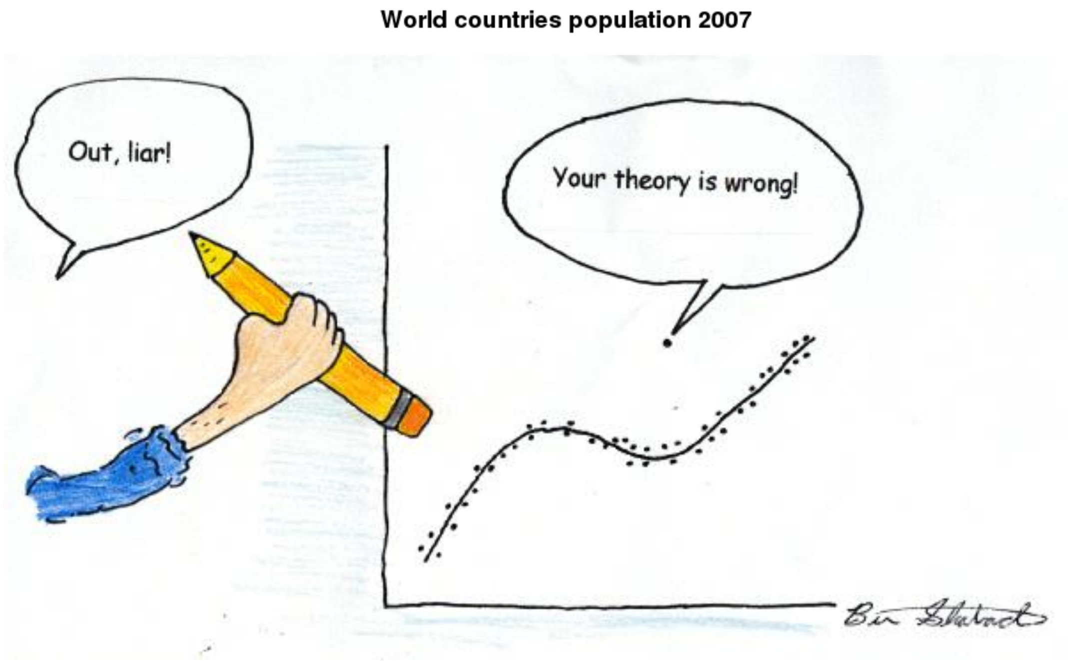
What should you do if you have an outlier in your data?

A: See if you can understand what is causing it!

- If it’s an error, delete the point
- If it’s a real value, make sure it is not having a bit effect on your conclusions, and/or use resistant statistics

Is the mean and/or median is resistant?

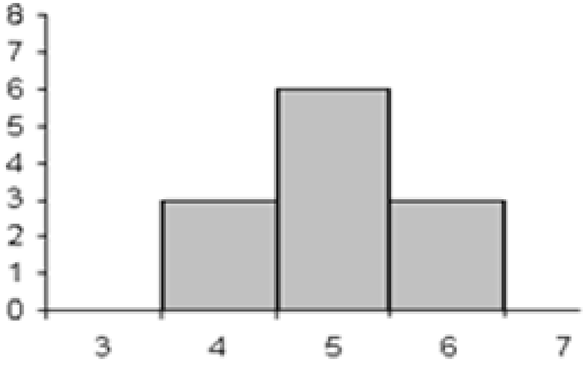
- A: The median is resistant when the mean is not



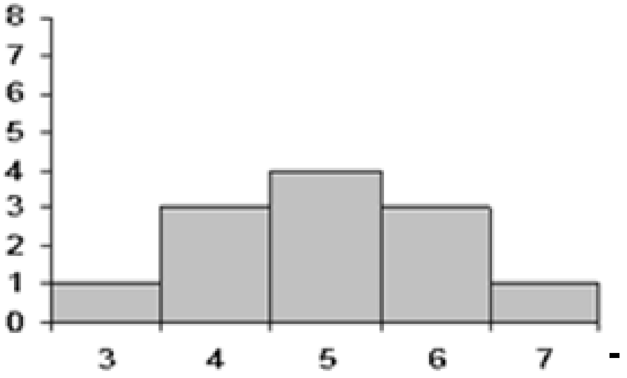
The standard deviation: a measure of spread

The **standard deviation** (for a quantitative variable) is a measure of the spread of the data

Smaller standard deviation

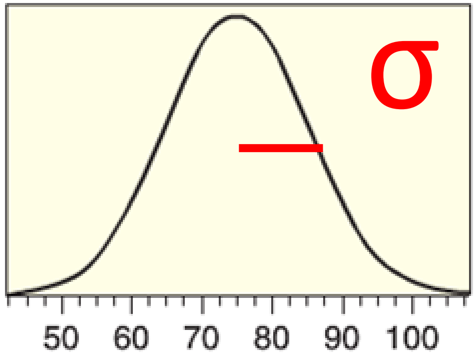


Larger standard deviation

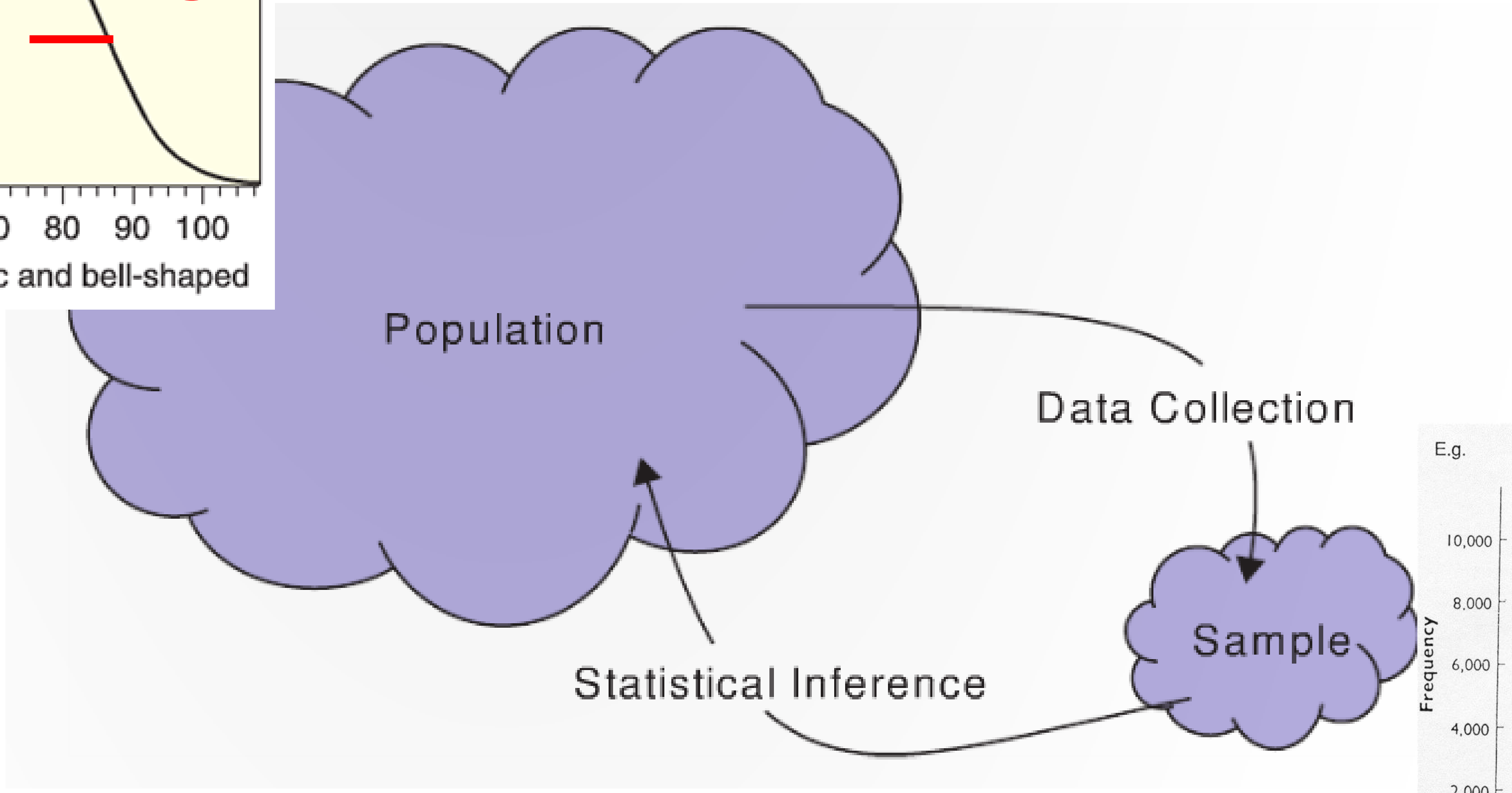


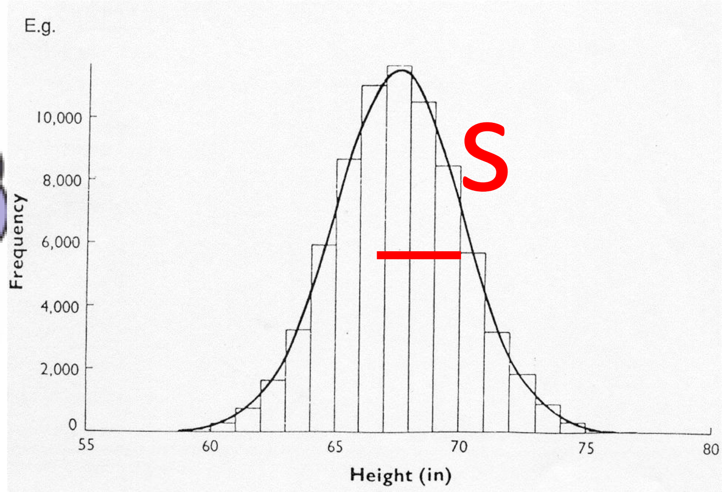
It gives a rough estimate for a typical distance a point is from the center

Population and sample standard deviation



(c) Symmetric and bell-shaped





The standard deviation

The standard deviation can be computed using the following formula:

$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

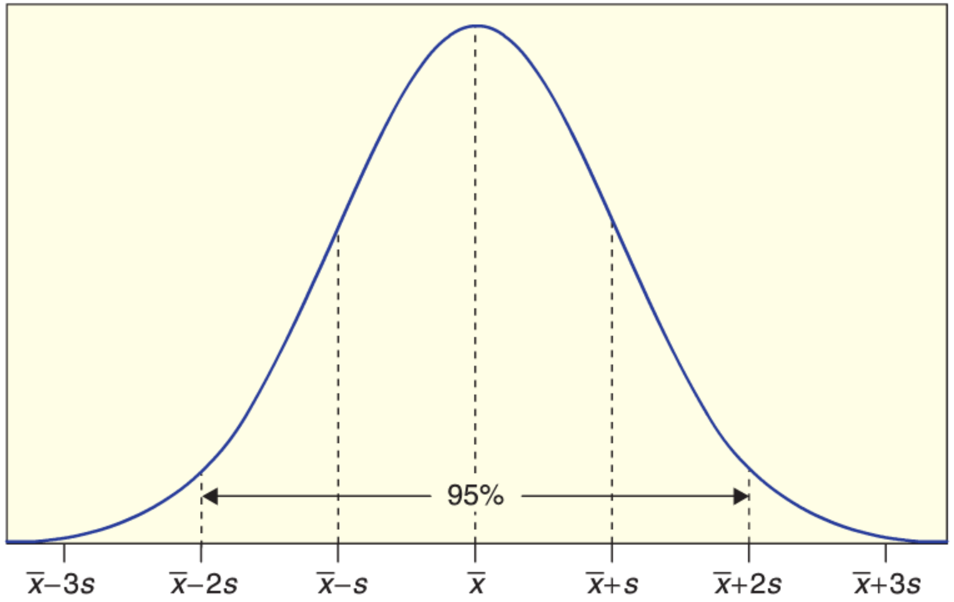
Part 1: Calculate the mean and standard deviation for the number of hot dogs eaten!

The 95% rule for *normal distributions*

A **normal distribution** is a common distribution that is symmetric and bell shaped

If a distribution of data is approximately normally distributed, about 95% of the data should fall within two standard deviations of the mean

i.e., 95% of the data is in the interval: $\bar{x}$ -2s to $\bar{x}$ +2s



The 95% rule for *normal distributions*

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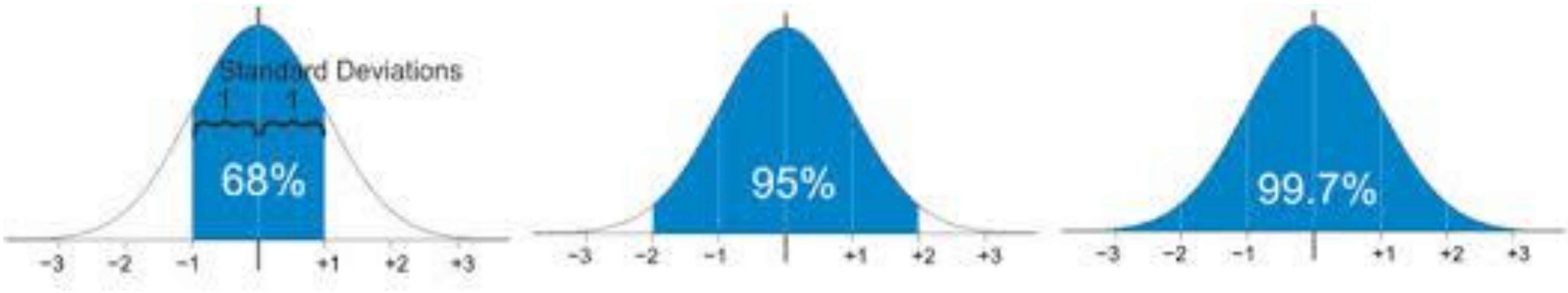
Example: IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

Question: what is the range of values that the middle 95% of IQ scores fall in?

Answer: (100 – 30) to (100 + 30), 95% of IQ scores are in the range 70 to 130



z-Scores

The z-scores tells how many standard deviations a value is from the mean

- i.e., how far away a point x_i is from $\bar{x}$ in a way that is independent of the units of measurement

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

Which Accomplishment is most impressive?

LeBron James is a basketball player who had the following statistics in 2011:

- Field goal percentage (FGPct) = 0.510
- Points scored = 2111
- Assists = 554
- Steals = 124

The summary statistics of the NBA in 2011 are given below

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

	Mean	Standard Deviation
FGPct	0.464	0.053
Points	994	414
Assists	220	170
Steals	68.2	31.5

Relative to his peers, which statistic is most and least impressive?

Which Accomplishment is most impressive?

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- Field goal percentage (FGPct) = 0.510
- Points scored = 2111
- Assists = 554
- Steals = 124

The summary statistics of the NBA in 2011 are given below

z	=	$(x - \bar{x}) / s$		
Z-score FGPct	=	(0.510 - 0.464)/0.053	=	0.868
Z- score Points	=	(2111 994)/414	=	2.698
Z-score Assists	=	(554 – 220)/170	=	1.965
Z-score Steals	=	(124 - 68.2)/31.5	=	1.771

Percentiles (quantiles)

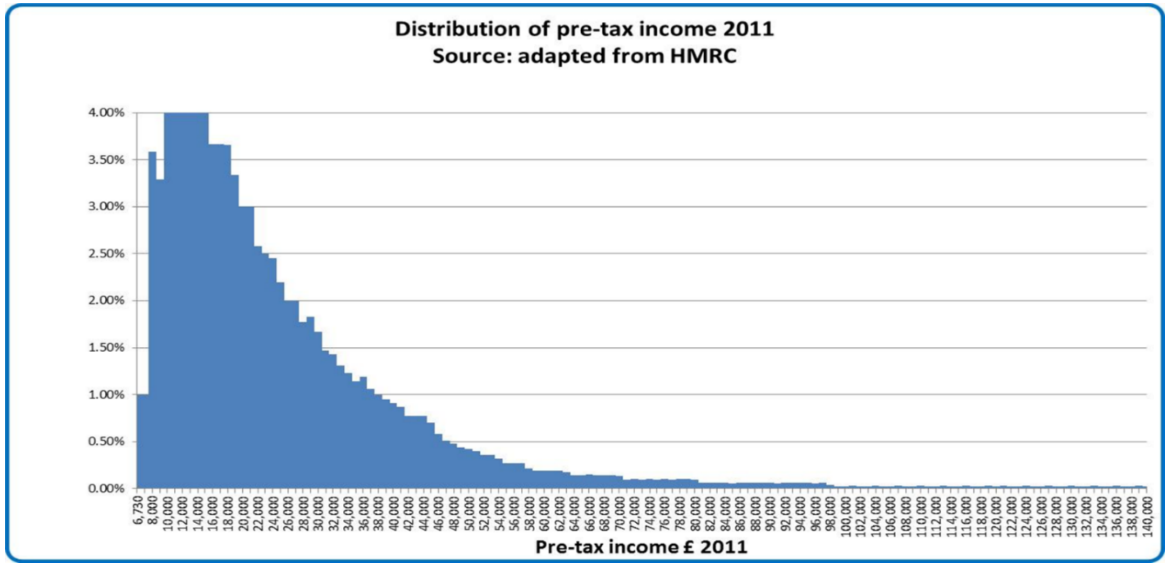
The **P^{th} percentile** is the value of a quantitative variable which is greater than P percent of the data

The income distribution is shown below. What are the 25^{th} and 90^{th} percentiles?

Income distribution

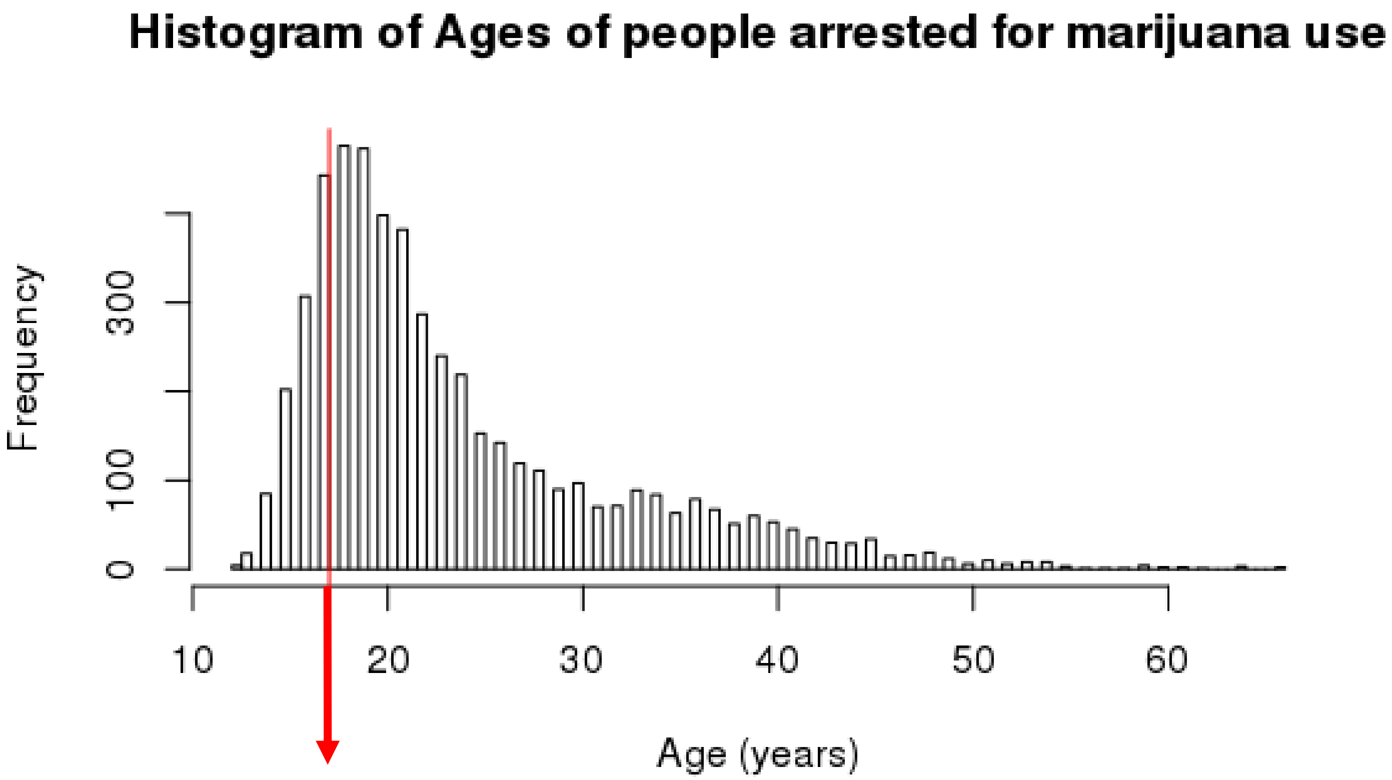
25^{th} percentile = \$27,794

95^{th} percentile = \$113, 820



R: `quantile(v, .95)`

Quantiles: age of marijuana arrests in Toronto



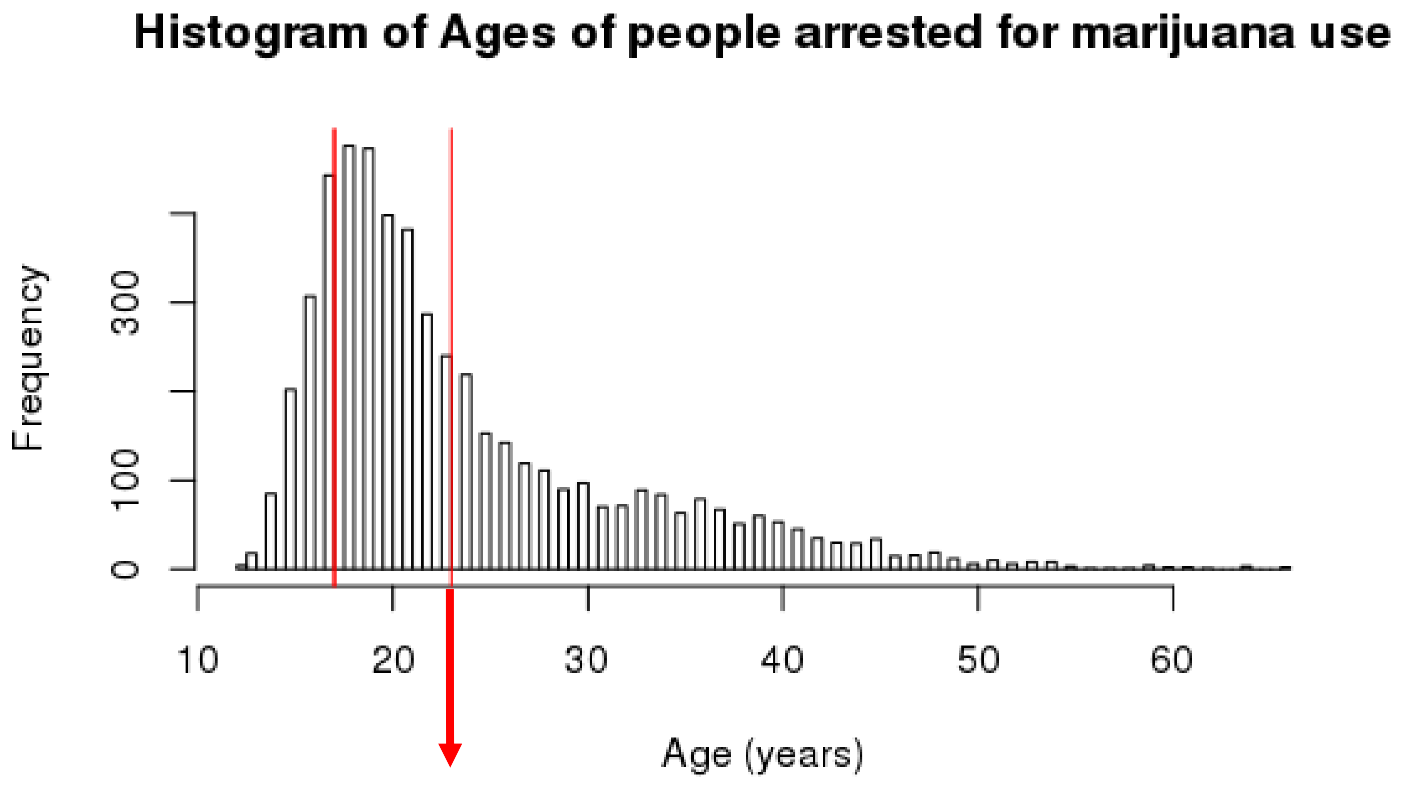
20th percentile value is 17

i.e., 20% of the arrests were of ages 17 or less

> library(carData) # load the data

> quantile(Arrests$age, .2)   # get the 20th percentile value from a vector of ages of arrests

Quantiles: age of marijuana arrests in Toronto



60th percentile value is 23

i.e., 60% of the arrests were of ages 23 or less

> quantile(Arrests$age, c(.2, .6)) # get the 20th and 60th percentile values from a vector of ages of arrests

Five Number Summary

Five Number Summary = (minimum, Q_1, median, Q_3, maximum)

Q_1 = 25th percentile (also called 1st quartile)

Q_3 = 75th percentile (also called 3rd quartile)

Roughly divides the data into fourths

Range and Interquartile Range

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

Hot dog example

Part 2: For the hot dog data calculate:

- The 5 number summary
- The range
- Interquartile range

Cheat sheet:

Five Number Summary = (minimum, Q_1, median, Q_3, maximum)

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

Q_1 = 25th percentile, Q_3 = 75th percentile

Answer in R: `fivenum(v)`

Detecting of outliers

As a rule of thumb, we call a data value an **outlier** if it is:

Smaller than: $Q_1 - 1.5 * IQR$

Larger than: $Q_3 + 1.5 * IQR$

What is the range that a value would be called an outlier in the hot dog data?

Are there any outliers in the hot dog data?

Boxplots

A **boxplot** is a graphical display of the 5 number summary and consists of:

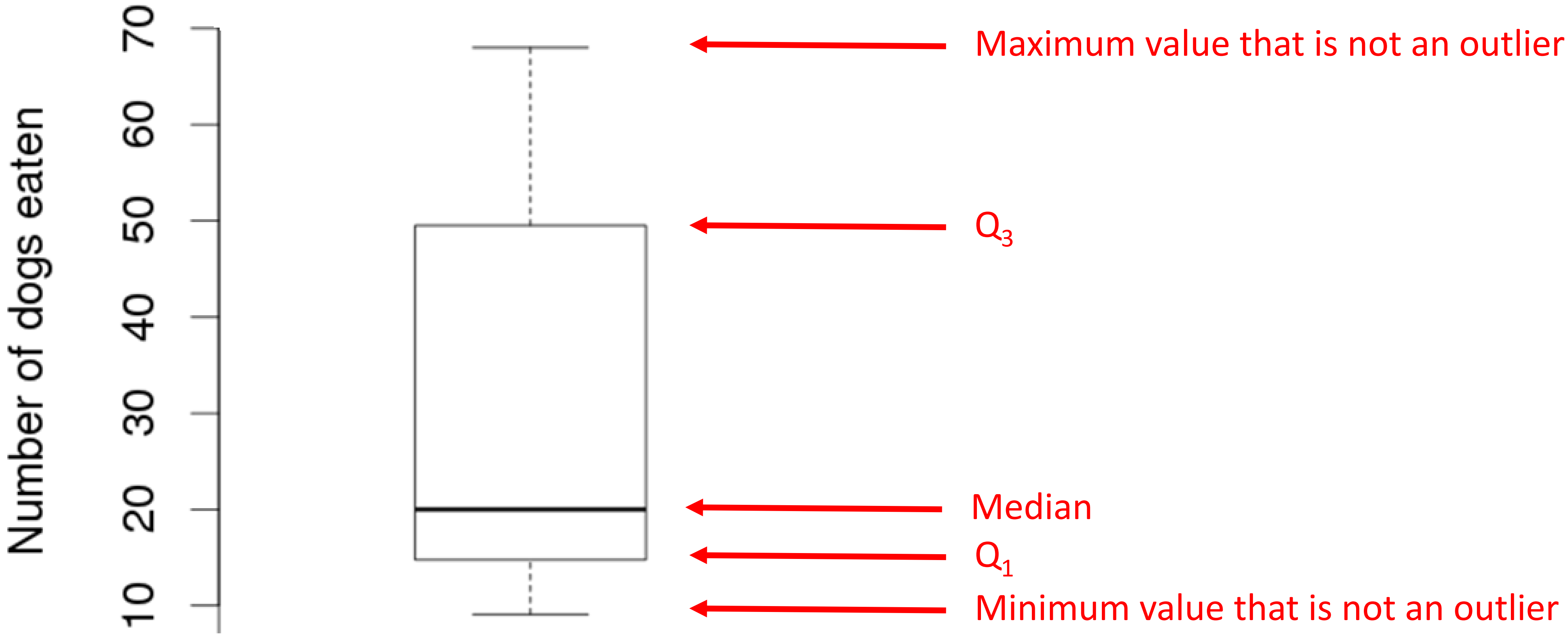
1. Drawing a box from Q_1 to Q_3

2. Dividing the box with a line (or dot) drawn at the median

3. Draw a line from each quartile to the most extreme data value that is not and outlier

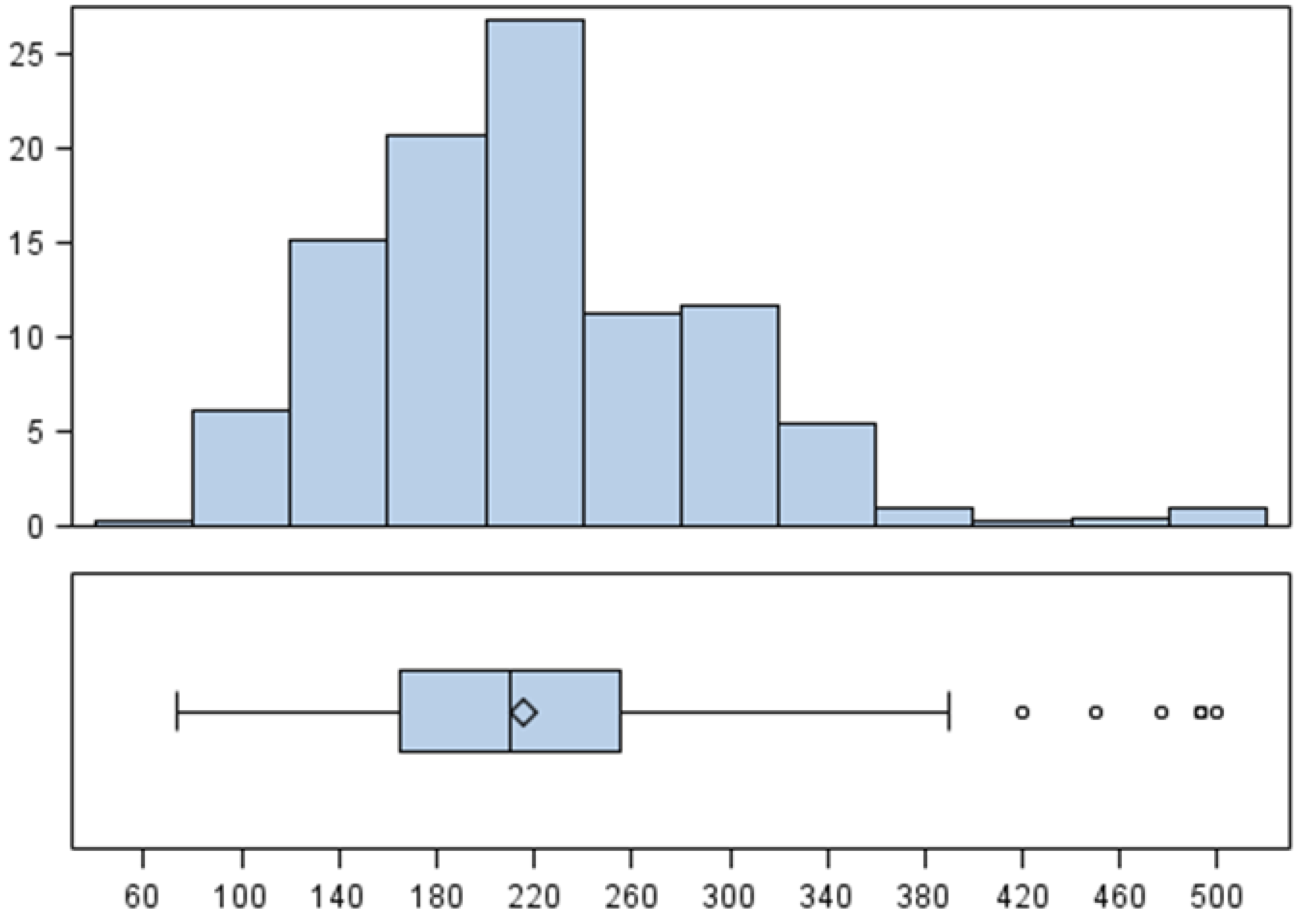
4. Draw a dot/asterisk for each outlier data point.

Box plot of the number of hot dogs eaten by the men's contest winners 1980 to 2010



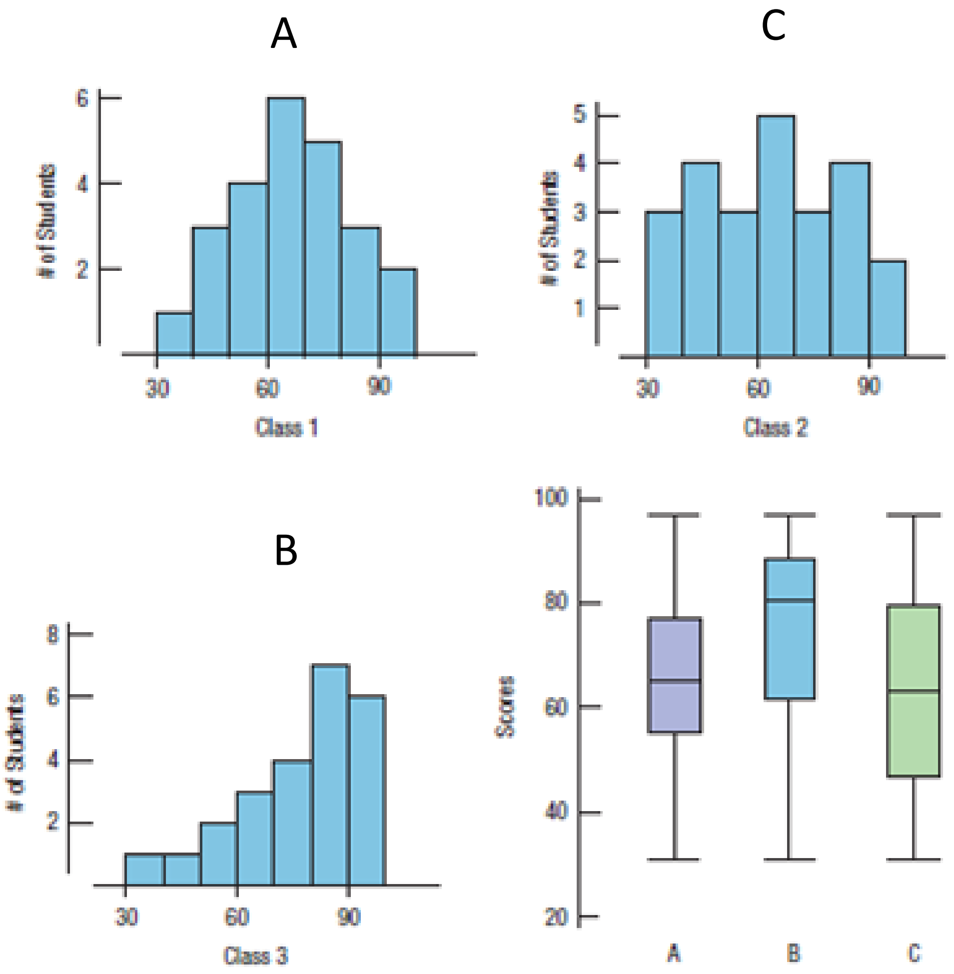
R: `boxplot(v)`

Box plots extract key statistics from histograms

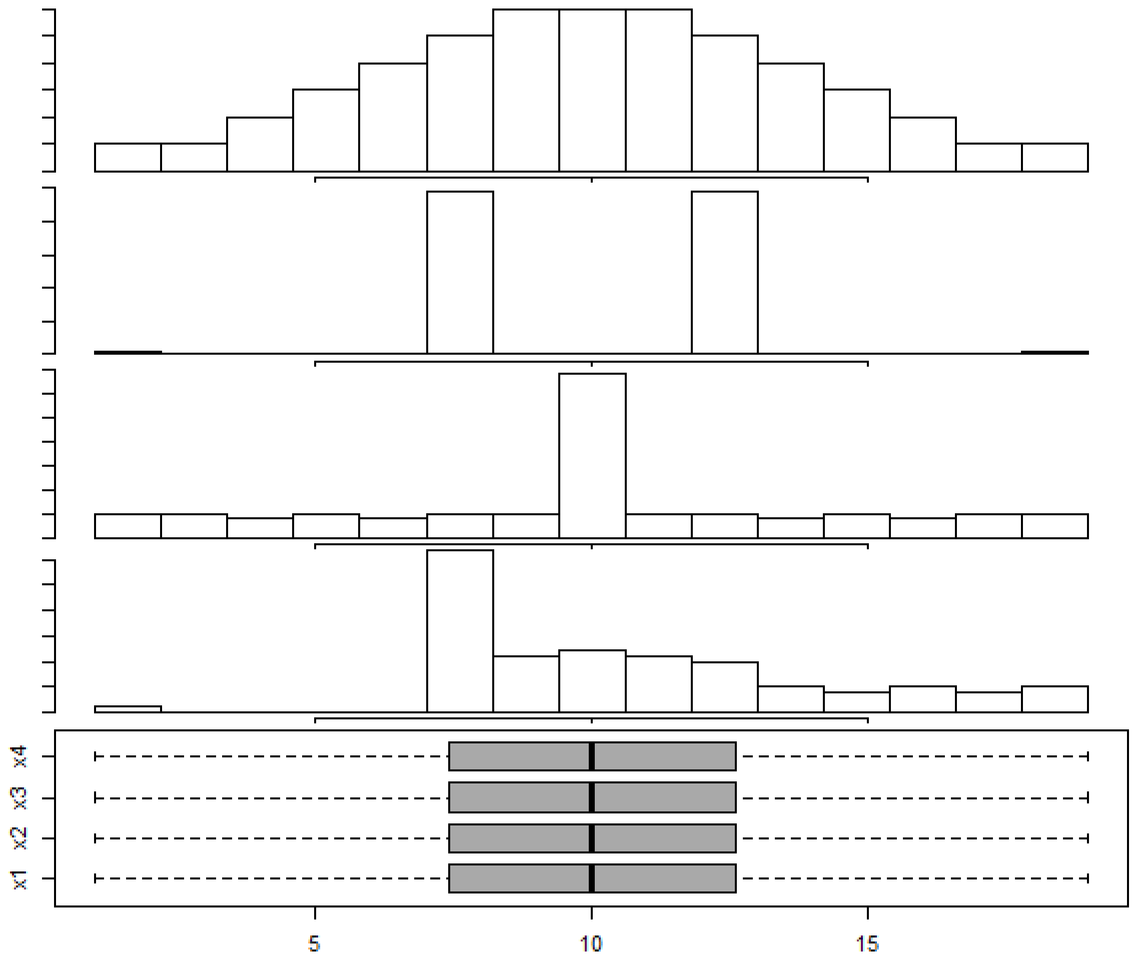


Box plots extract key statistics from histograms

Question: which Boxplot goes with which histogram?



Boxplots don’t capture everything

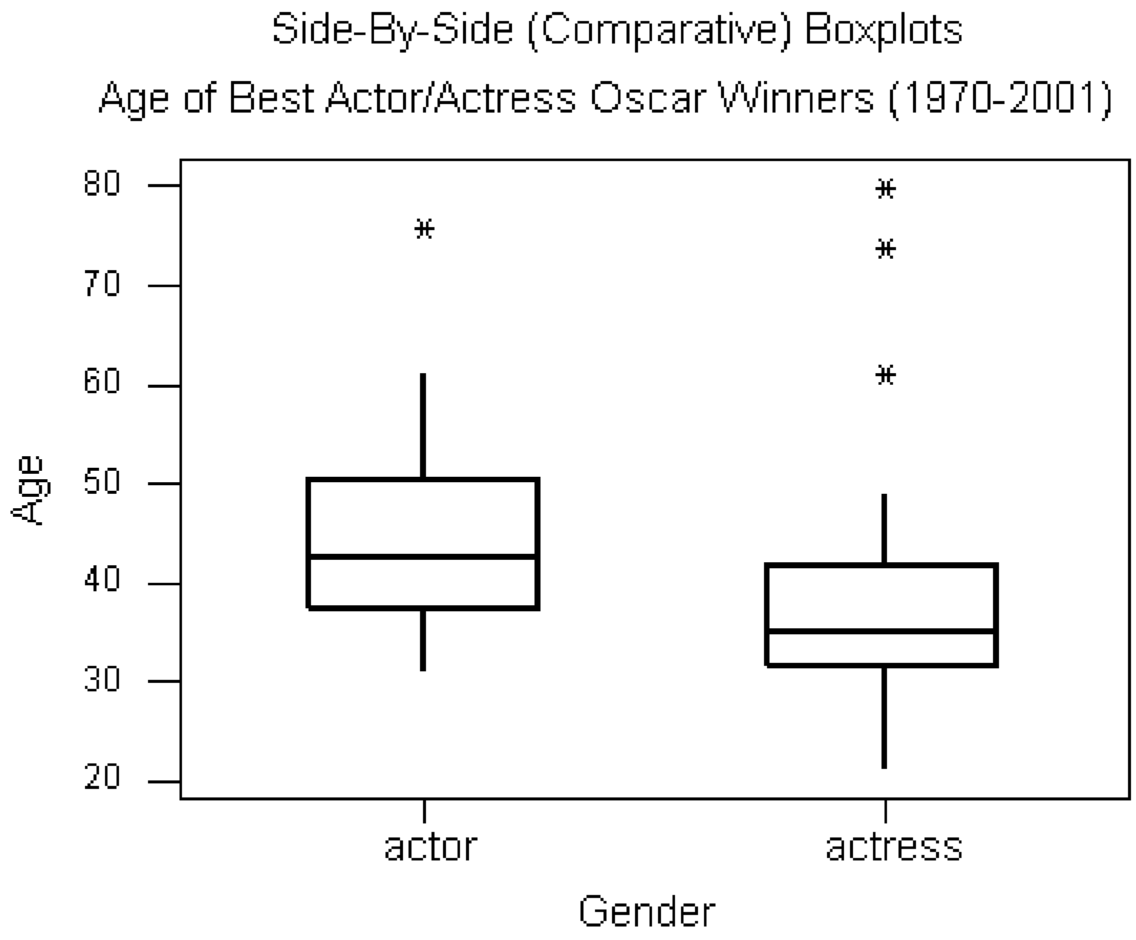


Comparing quantitative variables across categories

Often one wants to compare quantitative variables across categories

Side-by-Side graphs are a way to visually compare quantitative variables across different categories.

Side-by-side boxplots



Side-by-size boxplots in R

```
> boxplot(v1, v2,                          # compare two vectors v1 and v2
     names = c("name 1", "name 2"),        # labels below each boxplot
      ylab =  "y-axis name"                # y-axis label name
   )
```

Try it yourself, create histograms and boxplots for this data:

```
> load("/home/shared/intro_stats/cs206_data/diff_distribution_same_boxplot.Rda")
> boxplot(x1, x2, x3, x4)
```

Concepts/statistics for summarizing quantitative data

z-scores show how many standard deviations a point is from the mean

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

Quantiles show the value ***x,*** such that a fixed proportion of the data is less than ***x***

- e.g., what is the value *x,* such that 20% of the data is less than *x*

Five Number Summary give key summary statistics of a data sample

- minimum, Q_1, median, Q_3, maximum

A **boxplot** is a visualization of the five number summary

- Side-by-side boxplots allow you to compare key summary statistics

Summary of R

We can compute a z-score for a value x, and a vector of values v using:

```
> the_mean <- mean(v)
> the_sd <- sd(v)
> the_zscore <-   (x - the_mean)/the_sd
```

We can compute quantiles using the quantile() function:

- `> quantile(v, .2)` or `quantile(v,  c(.25, .4))`

We can compute a five number summary using the fivenum() function:

```
> fivenum(v)
```

We can compute boxplots using the boxplot() function

```
> boxplot(v)
```

Worksheet 3

Log on to asterius and type:

```
> source("/home/shared/intro_stats/cs206_functions.R")
> get_worksheet(3)
```

Due, Sunday September 30th at 11:59pm

Use Slack for questions – and answer questions posted by others

Worksheet is a little harder than the previous ones so get started early!